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Question Paper Code : 10878

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2025.

Fourth Semester

Computer and Communication Engineering

MA 8451 — PROBABILITY AND RANDOM PROCESSES

(Common to Electronics and Communication Engineering / Electronics and Telecommunication Engineering)

(Regulations 2017)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Which one of these variables is a continuous random variable?
 - (a) The time it takes a randomly selected student to complete an exam
 - (b) The number of tattoos a randomly selected person has
 - (c) The number of women taller than 68 inches in a random sample of 5 women
 - (d) The number of correct guesses on a multiple choice test.
2. If X follows an exponential distribution with parameter 1, find $P(|X| < 1)$.
3. State Central limit theorem.
4. If the pdf of X is $f_X(x) = 2x$, $0 < x < 1$, find the pdf of $Y = 3X + 1$.
5. Prove that Poisson process is a Markov process.
6. Define a random process and given an example of a random process.
7. State any two properties of power spectral density function.
8. What is the difference between auto correlation and cross-correlation functions?
9. What is a linear time-invariant system?
10. State the relation between input and output auto correlation functions in LTI systems.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) An inspector working for a manufacturing company has a 99% chance of correctly identifying defective items and a 0.5% chance of incorrectly classifying a good item as defective. The company has evidence that its line produces 0.9% of nonconforming items. If an item selected at random is classified as non-defective, what is the probability that it is indeed good? (8)
- (ii) If a random variable X has the MGF $M_x(t) = \frac{3}{3-t}$, obtain the standard deviation of X . (8)

Or

- (b) (i) The number of content changes to a Web site follows a Poisson distribution with a mean of 0.25 per day.
- (1) What is the probability of two or more changes in a day? (2)
 - (2) What is the probability of no content changes in five days? (2)
 - (3) What is the probability of two or fewer changes in five days? (2)
 - (4) What is the probability of more than one change in four days? (2)
- (ii) The fill volume of an automate filling machine used for filling cans of carbonated beverage is normally distributed with a mean of 12.4 fluid ounces and a standard deviation 0.1 fluid ounce.
- (1) What is the probability that a fill volume is less than 12 fluid ounces? (1)
 - (2) If all cans less than 12.1 or more than 12.6 ounces are scrapped, what proportion of cans is scrapped? (1)
 - (3) Determine specifications that are symmetric about the mean that include 99% of all cans. (2)
 - (4) At what value should the mean be set so that 99.9% of all cans exceed 12 fluid ounces. (2)
 - (5) At what value should the mean be set so that 99.9% of all cans exceed 12 fluid ounces if the standard deviation can be reduced to 0.05 fluid ounce? (2)

12. (a) The joint probability mass function of two random variables X and Y is given by $P_{XY}(x, y) = \begin{cases} k(2x + y), & x = 1, 2; y = 1, 2 \\ 0, & \text{otherwise} \end{cases}$ where k is a constant

- (i) What is the value of k ? (2)
- (ii) Find the marginal PMFs of X and Y (4)
- (iii) Determine $E(X)$, $E(Y)$, $V(X)$ and $V(Y)$ (4)
- (iv) Are X and Y independent? (3)
- (v) Find the covariance of X and Y . (3)

Or

- (b) If the joint density function of (X, Y) is given by $f(x, y) = 2 - x - y$, $0 \leq x, y \leq 1$, find $E(X)$, $E(Y)$, $Var(X)$, $Var(Y)$ and Correlation of X and Y .

13. (a) (i) The process $\{X(t)\}$ whose probability distribution under certain condition is given by $P\{X(t) = n\} = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1, 2, \dots \\ \frac{at}{1+at}, & n = 0 \end{cases}$

Show that it is stationary. (8)

- (ii) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is
 - (1) More than 1 minute (2)
 - (2) Between 1 minute and 2 minute (3)
 - (3) 4 minute or less. (3)

Or

- (b) (i) The transition probability matrix of the Markov chain with three states 0, 1, 2 is $P = \begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$ and the initial distribution is $(1/3 \ 1/3 \ 1/3)$. Find $P(X_2 = 2)$ and $P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$. (8)
- (ii) A stochastic process is described by $X(t) = A \sin t + B \cos t$, where A and B are independent random variables with zero mean and equal standard deviations. Show that the process is stationary. (8)

14. (a) If $\{X(t)\}$ and $\{Y(t)\}$ are independent WSS processes with zero means, find the auto correlation function of $\{Z(t)\}$, when

(i) $Z(t) = a + bX(t) + cY(t)$. (8)

(ii) $Z(t) = aX(t)Y(t)$. (8)

Or

- (b) Find the power spectral density function of the random process $\{X(t)\}$, if $E\{X(t)\} = 1$ and $R_{xx}(\tau) = 1 + e^{-|\tau|}$. (16)

15. (a) Let $X(t)$ be a zero-mean WSS process with $R_X(\tau) = \exp(-|\tau|)$. $X(t)$ is input to an LTI with $H(f) = \begin{cases} \sqrt{1 + 4\pi^2 f^2} & \text{if } |f| < 2 \\ 0 & \text{otherwise} \end{cases}$. Let $Y(t)$ be the output. Find $E[X(t)]$, $R(t)$ and $E[X(t)^2]$. (16)

Or

- (b) Let $X(t)$ be a random process with mean function $\mu_x(t)$ and autocorrelation function $R_X(s, t)$ ($X(t)$ is not necessarily a WSS process). Let $Y(t)$ be given by $Y(t) = h(t) * X(t)$. Where $h(t)$ is the impulse response of the system? Show that (16)

(i) $\mu_Y(t) = \mu_X(t) * h(t)$

(ii) $R_{XY}(t_1, t_2) = h(t_2) * R_X(t_1, t_2) = \int_{-\infty}^{\infty} h(\alpha) R_X(t_1, t_2 - \alpha) d\alpha$.